Public Debt in a Growing Economy

And Implications for the Nigerian Case

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Main Source of Presentation

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Wenzel, H.-Dieter (2006). Public Finance (Öffentliche Finanzen). Unpublished German Script from Bamberg University, Bamberg.

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1. Government Instruments

in a closed economy:

D	Deficit
B	Debt
T	Taxes
G	Expenditures
I^{pub}	Public Investment

2. Capital Formation (1 of 2)

$$S^{pr} = I^{pr} + D$$

$$\dot{K} = I^{pr}$$

$$\Rightarrow \quad \dot{K} = S^{pr} - D$$

$$S^{pr} = s(Y - T + rB) \qquad (r \equiv rental\ rate\ of\ capital)$$

$$D = G - T + rB$$

$$\Rightarrow \quad \dot{K} = s(Y - T + rB) - D$$

2. Capital Formation (2 of 2)

$$Y = F(K, L)$$ Production Function

$$k := \frac{K}{N} \Rightarrow y := \frac{Y}{N} = F(\frac{K}{N}, 1) = f(k)$$

$$I^{pub} = \kappa G, \quad 0 \le \kappa \le 1 \Rightarrow \begin{cases} \dot{K} = I^{pr} + I^{pub} \\ \dot{K} = s\left(Y - T + rB\right) - D + \kappa G \end{cases}$$

$$\frac{\dot{N}}{N} =: n, \quad \frac{\dot{K}}{N} = \dot{k} + nk \Rightarrow \boxed{\dot{k} = s(f - t + rb) - d + \kappa g - nk}$$

3. Deficit Financing

$$\dot{B} = D$$

$$\frac{\dot{B}}{N} = b + nb \quad \Rightarrow \quad \dot{b} = d - nb$$

$$\Downarrow$$

$$\boxed{\dot{b} = g - t + (r - n)b}$$

4. Dynamics

Basic Model with per capita tax **t** and per capita expenditures **g**

$$\boxed{\begin{aligned}\dot{k} &= sf(k) + (1-s)t - (1-s)br - 1(-\kappa)g - nk \\ \dot{b} &= g - t + (r-n)b\end{aligned}}$$

$r = f'(k)$ $\quad$ $r \equiv$ *rental rate of capital*

$\equiv$ *marginal product of capital (from profit max.)*

5. Steady States

Steady State Conditions and Isoclines

$$\left.\begin{aligned} \dot{b} &= 0 \\ \dot{k} &= 0 \end{aligned}\right\} \quad \Rightarrow \quad \left\{\begin{aligned} bb:&\quad b = b(k) \\ kk:&\quad b = b(k) \end{aligned}\right.$$

6. Optimal Steady States
(Optimal means Consumption Maximizing)

$$f(k) = c(k) + nk$$

$$\max_{\{k\}} c(k) \Rightarrow f'(k) = n \quad (f''(k) < 0)$$

$$(Golden\ rule)$$

7. Stable Steady States (1 of 2)

Do there exist stable golden rule steady states with positive public debt?

To give an answer, we will use a geometrical approach to analyse these characteristics.

In advance, however, we have to specify the tax and expenditure frameworks at issue. We consider a **spending-scenario** with

(G1) constant per capita expenditure $g = \bar{g}$

(G2) constant expenditure-output ratio $\bar{g}_y = g / f$

and correspondingly a **tax-scenario** with

(T1) constant percapita tax $t = \bar{t}$

(T2) constant income tax rate $\bar{\tau} = t / (f + rb)$.

7. Stable Steady States (2 of 2)

Obviously, four different tax-expenditure combinations exist, which can be analysed.

We analyze only **two** of them. Analysis of the other two scenarios you will find in the paper:

Wenzel (2001). Growth Equilibria with Public Debt. Society and Economy in Central and Eastern Europe. Journal of the Budapest University of Economic Sciences and Public Administration. Bd. (Vol.) 23/1-2 S. 70-88. Budapest .

Firstly, we discuss the case of constant per capita expenditure (G1) and correspondingly constant per capita tax (T1).

Secondly, the case of constant expenditure-output ratio (G2) and correspondingly constant income tax rate (T2).

8. First Scenario (1 of 6)

Combining the expenditure scenario (G1) with the lump-sum tax case (T1) we get the basic model in the following form

$$\dot{k} = F^k(k, b, s, n, \kappa, g, t)$$

with $F^k \equiv s\big(f - t + f'b\big) - (g - t + f'b) + \kappa g - nk$

and $\dot{b} = F^b(k, b, s, n, \kappa, g, t)$

with $F^b \equiv g - t + (f' - n)b.$

8. First Scenario (2 of 6)

Isoclines of the stationary values of k and b which are implicitly defined by the condition

$$F^k(k,b,s,n,\kappa,g,t) = F^b(k,b,s,n,\kappa,g,t) = 0$$

and can be explicitly expressed by

$$\boxed{\begin{aligned} kk:\quad & b(k) = (sf - nk + (1-s)t - (1-\kappa)g)/(1-s)f' \\ bb:\quad & b(k) = (g-t)/(n-f'). \end{aligned}}$$

8. First Scenario (3 of 6)

Geometrically, a steady state is the intersection of the isoclines in the b-k phase diagram.

Therefore, we are able to describe the **stability** characteristics of a steady state through both the shapes and intersections of the **isoclines**

and the shapes of the solution **trajectories** given by the dynamics of the system.

8. First Scenario (4 of 6)

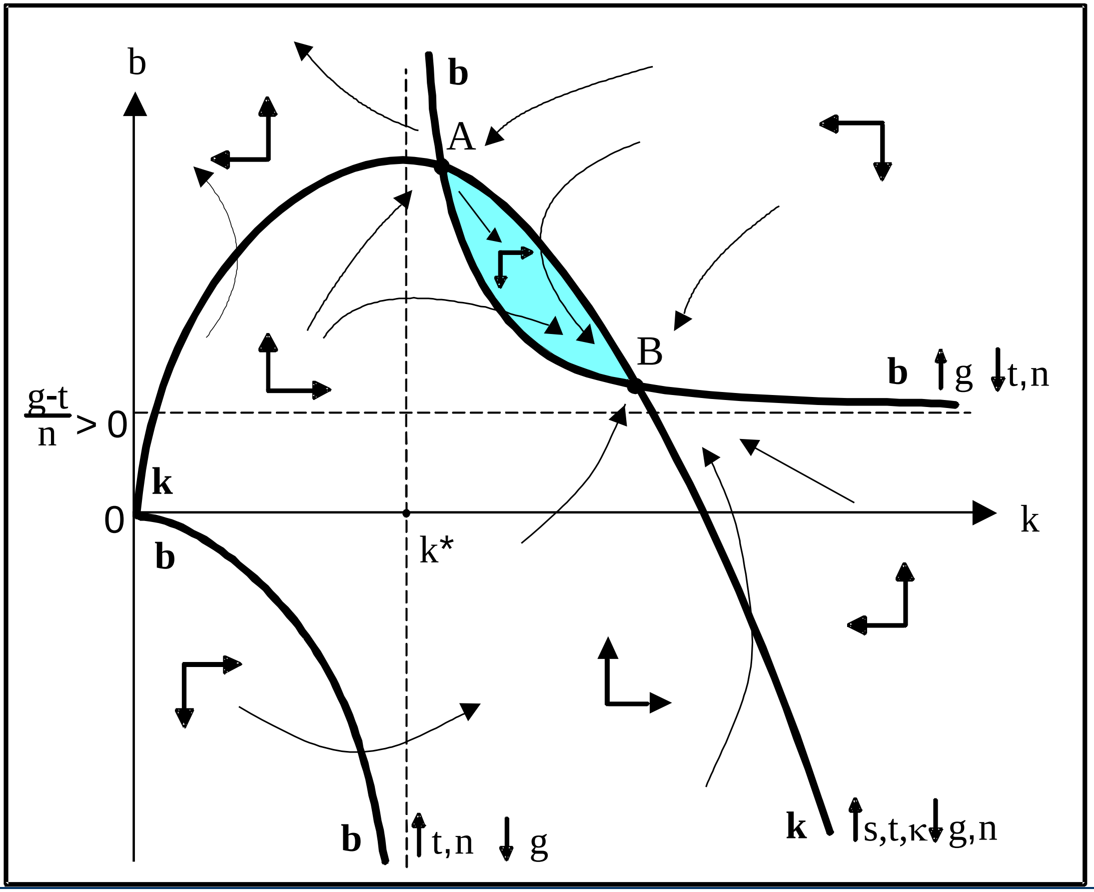
The shapes of the isoclines bb and kk are derived by simple algebraic and marginal analysis, using the properties of the production function. The condition

$$f'(k^*) = n$$

implicitly defines the **golden rule solution** with capital intensity k*, the bb-isocline obviously having a node at point $k = k^*$. We further assume parameter values that determine a positive slope of kk for values of $k > 0$ near the origin. However, these will not be crucial for our analysis.

8. First Scenario (5 of 6)

Lump-sum taxation t, constant per-capita expenditures g and positive primary deficit (g-t).



8. First Scenario (6 of 6)

Proposition 1: With lump-sum tax *t*, constant government per capita expenditures *g*, and **positive** primary deficit *g-t*, it follows

a) As long as the private savings rate is sufficiently high, there exist two steady states with **positive** government debt, given that the parameters g, t and have been chosen appropriately.

b) The steady state B with higher capital intensity is local asymptotically **stable**. The steady state A is a saddle point, and the origin is **unstable**.

c) All non trivial steady states are dynamically **inefficient**, and there exists no steady state at the capital intensity k* with maximal consumption.

9. Second Scenario (1 of 4)

Combining the expenditure (G2) with the tax rule (T2) we get the basic model in the modified form:

$$\dot{k} = F^k(k, b, s, n, \kappa, g_Y, t)$$

with $F^k \equiv \left[1-(1-s)(1-\tau)-g_y(1-\kappa)\right] f-(1-s)(1-\tau) f' b-nk$

and $$\dot{b} = F^b(k, b, s, n, \kappa, g_Y, t)$$

with $F^b \equiv \left(g_y-\tau\right) f+\left(f'(1-\tau)-n\right) b$

9. Second Scenario (2 of 4)

Hence, the equations for the **isoclines** are

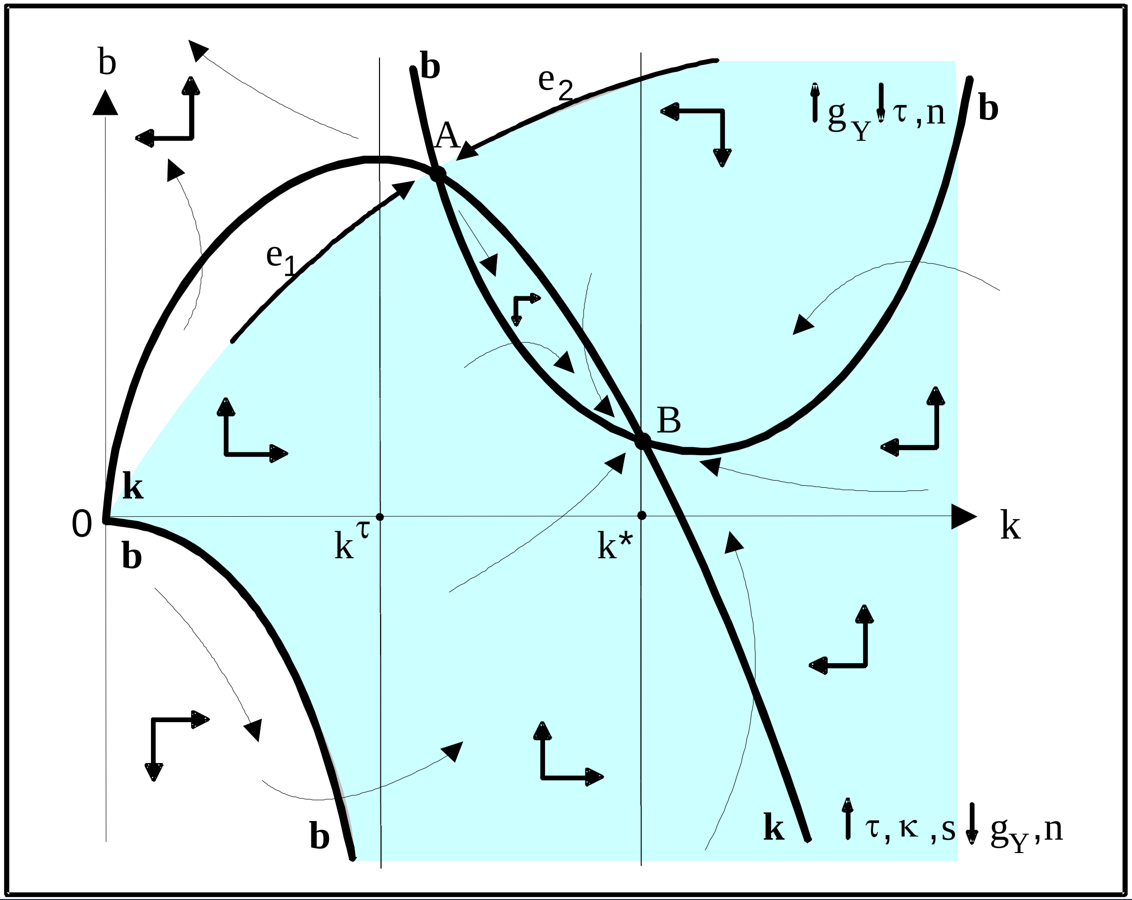
$$kk: \quad b(k) = \frac{\left[1-(1-s)(1-\tau)-g_y(1-\kappa)\right] f - nk}{(1-s)(1-\tau) f'}$$

$$bb: \quad b(k) = \frac{(g_y-\tau) f}{n - f'(1-\tau)}.$$

In the **phase diagram**, we have to distinguish the cases $g_y \lesseqgtr \tau$ explicitly, but we will discuss here the case only with **positive primary deficit**, $g_y - \tau > 0$.

9. Second Scenario (3 of 4)

Constant income tax rate τ, constant expenditure-output ratio g_y and positive primary deficit $(g_y - \tau)$



9. Second Scenario (4 of 4)

Theorem on Stable Steady States with Public Debt

In case of fixed expenditure-output ratios and income taxation with a fixed income tax rate it holds

1. **Stable** steady states with **positive** public debt require positive primary deficits.
2. A steady state with **positive** public debt will be **stable** if the slope of the bb-isocline **exceeds** the slope of the kk-isocline and if simultaneously the slope of the kk-isocline is **lower** than

$$F_b^b / F_b^k > 0.$$

Stability is used here in the meaning of local asymptotical stability.

Proof: *See Wenzel (2001).*

10. Conclusion (1 of 2)

We identified several steady states in the model of a closed economy with governmental activity and growth. Including the origin, there can exist up to **three** equilibria. The origin is the unstable solution, the steady state with the highest capital intensity is local asymptotically stable and the other one is a saddle point.

Depending on the kind of tax- and expenditure scenario, steady states could be characterized by over- or under-capitalisation and positive or negative public debt solutions.

10. Conclusion (2 of 2)

We found various **stable** growth equilibria with **positive** public debt. But only one steady state was at the same time **consumption-maximizing**. Any other stable steady state with positive public debt was associated with overcapitalisation.

Only in case of income taxation with a constant rate, dynamically inefficient solutions could be prevented.

And in addition, the combination of income taxation (with a **constant rate**) with an expenditure policy with **constant expenditure-output ratio** (instead of constant per capita value) may lead to **stable golden rule solutions with positive public debt.**

11. The Nigerian Case (1of 5)

1. Nigerian Debt Data
2. **Sustainable** Public Deficits
3. **Real** Public Debt and **Real** Deficits
4. Sustainability and **Primary Deficits**

11. The Nigerian Case (2 of 5)

Table: Debt Nigeria (years end in billion USD)

Year	External Debt	Domestic Debt		Debt Stock
		Federal	States	
2005	Debt Relief	.	.	.
2006	3,5	13,8	.	17,3
2007	3,7	18,6	.	22,2
2008	3,7	17,7	.	21,4
2009	3,9	21,9	.	25,8
2010	4,6	30,5	5,1	40,1
2011	.	.	.	.
Mär 12	5,6	39,2	.	**44,8**

Quelle: Print Media

11. The Nigerian Case (3 of 5)

- Sustainability and Public Deficits
- Public Debt (2012) = 76,5 billion USD (estimated)
- GDP (2012)=270 billion USD (estimated)
- Debt Ratio b=28%
- Deficit Ratio d=2,85%
- Inflation Rate π=10%
- Real Growth Rate GDP (2012) g=7%
- If debt ratio b remains constant, then there is room for the **deficit ratio d** with
  - $d=b(\pi+g)=4{,}76\%$

11. The Nigerian Case (4 of 5)

- Real Deficits and the Inflation Tax
- The nominal value **n** of the real deficit is given by

$$n=d-\pi b$$

- If d=2,85%, π=10% and b=28% then

$$n=0.05\%$$

- That means, in the Nigerian case for the year 2012, the nominal value of the real deficit will be only 0,135 billion USD, or 135 million USD

11. The Nigerian Case (5 of 5)

- Sustainability and primary deficits with ratio p
- Government Budget Restraint in **GDP ratio** terms

$$d=p+(r+\pi)b$$

- Condition for time constant debt ratios

$$d=b(\pi+g)$$

- Both together deliver the ratio of the primary deficit under the assumption of time constant debt ratios

$$p=b(g-r)$$

- For the Nigerian case with real growth rate $g=7\%$, real interest rate $r=5\%$, and debt ratio $b=28\%$ we get a positive sustainable primary deficit of $p=0{,}56\ \%$ of GDP

12. World Debt Data

Country	Population in million	GDP in billion Euro	Debt/GDP in %	Debt in billion Euro
Germany	80	2500	80	2000
France	65	1800	90	1650
Italy	60	1400	120	1700
Spain	50	1000	70	700
Greece	10	230	170	360
Portugal	10	160	90	145
Ireland	5	160	110	180
USA	310	10000	100	10000
Japan	130	3500	229	8000
Nigeria	170	170	18	30

Thank You for Your Attention

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